PROTEUS DS

Wave drift load modelling

Documentation - Theory - Validation

DSA®

Last revised: August 3, 2017
Version: 3821

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DYNAMIC SYSTEMS ANALYSIS

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1 Introduction

1.1 Overview

This document describes the development of wave drift force modelling capabilities within ProteusDS. The slowly varying wave drift load is needed to model systems with low natural periods such as a large moored ship. These wave drift loads are a second order wave effect. In monochromatic waves, a body experiences a low magnitude mean (steady) wave drift load. The mean wave drift load arises from the combination of the first order wave loads and the first order body motions. The mean wave drift load can also be determined by direct pressure integration over the hull of the body [1]. When modelling non-linear Froude-Krylov forces, special care should be taken not to double count mean wave drift forces.

In bichromatic waves, the 2nd order wave drift forces are also slowly varying at a frequency equal to the difference between the two waves. To determine compute these forces, a 2nd order boundary element method (BEM) solver such as WAMIT 6.4S is required. However, using Newman's approximation, the slowly varying wave drift load can be approximated from the mean wave drift loads for deep water waves.

Figure 1 shows a harmonic analysis of the wave elevation, and surge and heave excitation forces acting on a fixed sphere in bi-chromatic waves. It shows peaks at both wave frequencies ω_1 and ω_2. It should some mean loading at zero frequency, and some slowly varying loading at a frequency of $|\omega_2 - \omega_1|$. Also shown are sum frequency loads at $2\omega_1$, $2\omega_2$, and $\omega_1 + \omega_2$. These can be important for tension-leg platforms where the natural frequency can be fairly high. However to date, ProteusDS does not handle the sum frequency forcing.

While these second order forces are small, they can be important depending on the natural frequency of moored structures. Even with sea state wave spectrum frequencies far from the moored system, the sum or difference frequencies resulting from second order excitation forces can approach some of the natural frequencies of the structure. Heave natural frequencies of tension leg platform systems can be susceptible to resonant excitation from sum frequency loading. Yaw natural frequencies of large single point moored structures can be susceptible to resonant excitation from difference frequency loading. In both instances, large amplitude motions can occur leading to large reaction loads from the mooring or other components of the floating system [3].

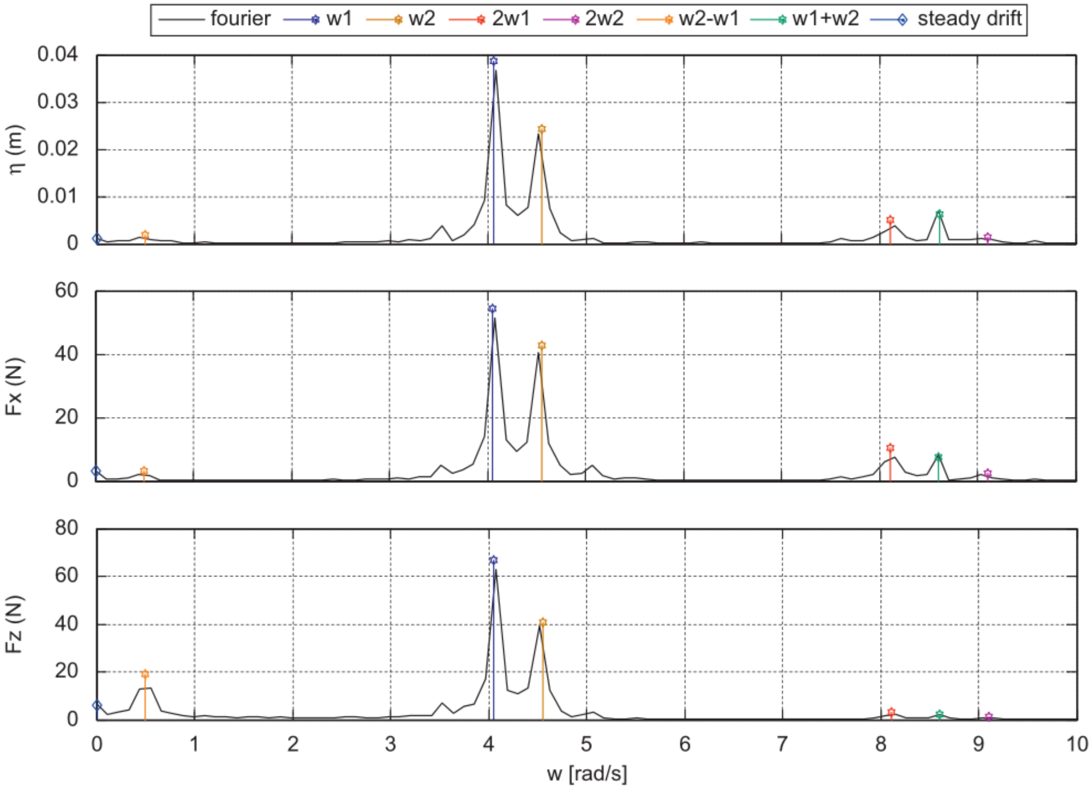

Figure 1: Harmonic analysis of the wave elevation, surge and heave excitation of a fixed sphere in bi-chromatic waves. Wave periods are 1.55s and 1.38s with corresponding amplitudes of 3.4cm and 3.0cm. This figure was taken from [2]

2 Modelling details

2.1 Overview

This section reviews background details on modelling mean and slowly varying wave drift forces. The details of these applicable formulas were implemented in ProteusDS.

2.2 Slowly varying wave drift loads

The 2nd order wave forces can be decomposed into three components: a mean component, a difference frequency component, and a sum frequency component. The difference and sum frequency components are a results of the interactions between pairs of waves and the body disturbance the presence of the body has on the wave field [3]. The mean and difference frequenciy components are generally called the mean wave drift load and slowly varying wave drift loads. This document only covers these 2nd order wave drift loads (mean and frequency difference).

These forces are generally computed by a BEM solver, which returns the forces in the form of a quadratic transfer functions (QTFs). These QTFs are imported by ProteusDS and used to apply mean and slowly varying wave drift loading.

The general formula used to model these mean and slowly varying wave drift loads are written as [1]:

$$\mathbf{F}_{wd} = \sum_{j=1}^{N}\sum_{k=1}^{N} A_j A_k \Re(\mathbf{Q}_{jk} e^{i(\phi_k - \phi_j)}) \tag{1}$$

where A_j and A_k are the amplitudes of wave component j and k, respectively, of an irregular sea state with N wave components, $e^{i(\phi)}$ is the complex number representation of a wave, $\Re(*)$ is the real component of the complex number $*$, and ϕ_j and ϕ_k are the current wave phase for wave component j and k, respectively. The wave phase is expressed as:

$$\phi = kX - \omega t + \epsilon \tag{2}$$

where $\mathbf{F}_{wd}$ is the length 6 wave drift load vector, k is the wave number, $\mathbf{Q}_{jk}$ is length 6 vector of complex numbers that represent the QTFs, X is the displacement from the origin along the wave heading vector, ω is the wave frequency and ϵ is the wave constant phase offset.

2.3 The QTF

The QTFs are 6 degree of freedom (DOF) vectors of coefficients that are a function of the frequency and the relative heading directions for each wave pair. It is 4 dimensional matrix of 6 DOF vectors of complex coefficients. It is dependent on both the heading and frequency of each wave in a pair.

$$\mathbf{Q}_{jk} = \mathbf{Q}(\theta_j, \theta_k, \omega_j, \omega_k) \tag{3}$$

where θ is the wave's relative heading.

For sea states which are composed of many wave segments, a very large database of QTF coefficients is required to capture each combination of wave frequencies and headings. The size of the database required to store these QTF coefficients grows rapidly with number of waves. Also, a second order potential flow solver like WAMIT 6.4S is required to compute the full QTF database for a structure.

The main diagonal of the QTF matrix and can be computed using some linear potential flow solvers like WAMIT 7.0 and NEMOH. The difference frequency, or the difference in the wave phases $\phi_k - \phi_j$, of two identical waves is zero. Therefore, the main diagonal coefficients produce the mean wave drift load from the corresponding single wave frequency.

2.4 Newman's approximation

Newman's approximation can be used to estimate the remaining QTF terms using only the main diagonal QTFs. A limitation is that the Newman approximation works well only in deep water conditions. Newman proposed that $\mathbf{Q}_{jk}$ can be approximated using $\mathbf{Q}_{jj}$, $\mathbf{Q}_{kk}$:

$$\Re(\mathbf{Q}_{jk}) = \Re(\mathbf{Q}_{kj}) = \frac{1}{2}(\Re(\mathbf{Q}_{jj}) + \Re(\mathbf{Q}_{kk})) \tag{4}$$

$$\Im(\mathbf{Q}_{jk}) = \Im(\mathbf{Q}_{kj}) = 0 \tag{5}$$

where $\Im(*)$ is the imaginary component of the complex number $*$. This is how ProteusDS uses Newman's approximation. ProteusDS will automatically use Newman's approximation if it finds that the QTFs that were imported only contain values for the main diagonal of the QTFs.

The imaginary component is always zero and the real component is formed using the arithmetic mean of $\mathbf{Q}_{jj}$ and $\mathbf{Q}_{kk}$. Alternatively, the geometric mean could be used instead:

$$\begin{array}{ll} \Re(\mathbf{Q}_{jk}) = \Re(\mathbf{Q}_{kj}) = \operatorname{sgn}(\mathbf{Q}_{jk})\sqrt{\Re(\mathbf{Q}_{jj})^2 + \Re(\mathbf{Q}_{kk})^2}, & if\, \operatorname{sgn}(\mathbf{Q}_{jk}) = \operatorname{sgn}(\mathbf{Q}_{kj}) \\ \Re(\mathbf{Q}_{jk}) = \Re(\mathbf{Q}_{kj}), = 0 & else \\ \Im(\mathbf{Q}_{jk}) = \Im(\mathbf{Q}_{kj}), = 0 & always \end{array} \tag{6}$$

Newman proposed a further approximation which turns equation 1 further turning it from a $O(N^2)$ operation to a $O(N)$ operation. However, this capability is not yet implemented in ProteusDS.

2.5 Unidirectional wave drift data

ProteusDS expects QTF data to be provided as a function of two different directions. ProteusDS will detect that no coefficients were provided where $\theta_j ! = \theta_k$ that the wave provided are uni-directional, i.e., coefficients are only provided where $\theta_j = \theta_k$. In this case, the QTF coefficients will be chosen using the mean heading between wave segments j and k:

$$\mathbf{Q}_{jk} = \mathbf{Q}(\theta_{mean}, \theta_{mean}, \omega_j, \omega_k) \tag{7}$$

Otherwise, the QTF coefficients are interpolated normally between available data points.

3 Manual

The wave drift loads need to be computed by a BEM solver such as WAMIT or NEMOH. First order BEM solvers can compute the mean wave drift loads. Then through Newman's approximation the mean wave drift loads can approximate the slowly varying wave drift loads for deep water. Second order BEM solvers can compute the full QTF and provide more accurately the slowly varying wave drift load.

BEM solvers will compute and provide a set of coefficients dependent on wave frequency and direction in their own output format. This output of coefficients form what is from here on referred to as a hydrodynamic coefficient database (HDDB). The HDDB must then be converted into a format that can be read by ProteusDS. DSA makes available a beta utility called "NEMOH/WAMIT converter" which converts the HDDB output by WAMIT or NEMOH into a format that can be read by ProteusDS. The HDDB is used by a ProteusDS RigidBody's RadDiffHydrodynamicModel feature. The NEMOH/WAMIT converter can be downloaded by choosing "Help > Visit Beta Utilities Page" from the menu bar in PST. The format of the HDDB required by ProteusDS is described in the ProteusDS Manual. A user could create their own converter to convert an HDDB from any BEM solver into a format that will be compatible with ProteusDS.

In the RadDiffHydrodynamicModel feature, wave drift loading can be enabled by setting the parameter “WaveDriftLoading” to 1. The loading will only be computed and applied if the frequency difference QTFs are provided.

The QTFs are a set 6DOF complex numbers that are dependent on the frequencies and headings of 2 separate waves. A full set QTFs can be quite large after accounting for all wave headings and directions and combinations thereof.

3.1 Interface changes

The following parameter is new to the RadDiffHydrodynamicModel feature:

- WaveDriftLoading

It is a flag that enables or disables wave drift loading for the model. When activated, wave drift loading effects are applied. These wave drift loads are computed from frequency difference QTF data exported by the BEM solver. Wave drift load modelling is currently in an experimental state. The hydrodynamic database of coefficients requires frequency difference QTF data for the loading to be applied. Sum frequency QTF data is not currently supported. If only the main diagonal of the QTF matrix is provided, ProteusDS will automatically apply Newman’s approximation. Note that Newman’s approximation degrades in shallow water. If only unidirectional wave pair QTF data is provided, ProteusDS with automatically take the average of the simulated wave pair’s headings in the wave drift load computation.

The following parameters are new parameters that have been added to the existing HDDB file format:

- QTFFrequencies
- QTFHeadings
- WaveDriftQTFMatrix

Documentation for these new parameters can be found in the ProteusDS manual. The documentation for these are provided here for convenience:

$QTFFrequencies

Description:	These wave frequencies that are referenced by index by the WaveDriftQTFMatrix.
Type:	matrix, variable number of columns, unordered, optional
Default:	$\left[\begin{array}{c} 0 \end{array}\right]$
Units:	rad/s

$QTFHeadings

Description:	These relative wave headings that are referenced by index by the WaveDriftQTFMatrix.
Type:	matrix, variable number of columns, unordered, optional
Default:	$\left[\begin{array}{c} 0 \end{array}\right]$
Units:	$Deg.$

$WaveDriftQTFMatrix

Description:		QTF coefficients for wave drift loading.
Type:		matrix of strings, optional
Default:		$\left[\begin{array}{cccccccccccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$
Column 0:		
	Name	Wave 1 frequency index
	Info	The index corresponding to the frequency of the 1st wave, defined in QTFFrequency, associated with this QTF.

Column 1:		
	Name	Wave 2 frequency index
	Info	The index corresponding to the frequency of the 1st wave, defined in QTFFrequency, associated with this QTF.
Column 2:		
	Name	Wave 1 heading index
	Info	The index corresponding to the heading of the 1st wave, defined in QTFHeadings, associated with this QTF.
Column 3:		
	Name	Wave 2 heading index
	Info	The index corresponding to the heading of the 2nd wave, defined in QTFHeadings, associated with this QTF.
Column 4:		
	Name	Real part of Surge QTF
	Info	The real part of the surge QTF for this set of wave frequencies and headings.
Column 5:		
	Name	Imaginary part of Surge QTF
	Info	The imaginary part of the surge QTF for this set of wave frequencies, headings.
Column 6:		
	Name	Real part of Sway QTF
	Info	The real part of the sway QTF for this set of wave frequencies and headings.
Column 7:		
	Name	Imaginary part of Sway QTF
	Info	The imaginary part of the sway QTF for this set of wave frequencies, headings.
Column 8:		
	Name	Real part of Heave QTF
	Info	The real part of the heave QTF for this set of wave frequencies and headings.
Column 9:		
	Name	Imaginary part of Heave QTF
	Info	The imaginary part of the heave QTF for this set of wave frequencies, headings.
Column 10:		
	Name	Real part of Roll QTF
	Info	The real part of the roll QTF for this set of wave frequencies and headings.
Column 11:		
	Name	Imaginary part of Roll QTF
	Info	The imaginary part of the roll QTF for this set of wave frequencies, headings.
Column 12:		
	Name	Real part of Pitch QTF
	Info	The real part of the pitch QTF for this set of wave frequencies and headings.
Column 13:		
	Name	Imaginary part of Pitch QTF
	Info	The imaginary part of the pitch QTF for this set of wave frequencies, headings.
Column 14:		
	Name	Real part of Yaw QTF
	Info	The real part of the yaw QTF for this set of wave frequencies and headings.
Column 15:		
	Name	Imaginary part of Yaw QTF
	Info	The imaginary part of the yaw QTF for this set of wave frequencies, headings.

4 Verification

4.1 Verification and validation test cases

The results described in this section are part of DSA's verification and validation testing suite. The results of 6 test cases are reported here (AT760-AT765). The first three cases validate the loads predicted by ProteusDS and WAMIT for a body in monochromatic waves against wave tank tests. The last three validate do the same but for bichromatic waves.

4.2 Setup

To verify the implementation of wave drift forces, the experimental results of two experiments were reproduced [2, 3]. The case consists of a fixed rigid body filleted cylinder. The model parameters are illustrated in Figure 2 and summarised in Table 1.

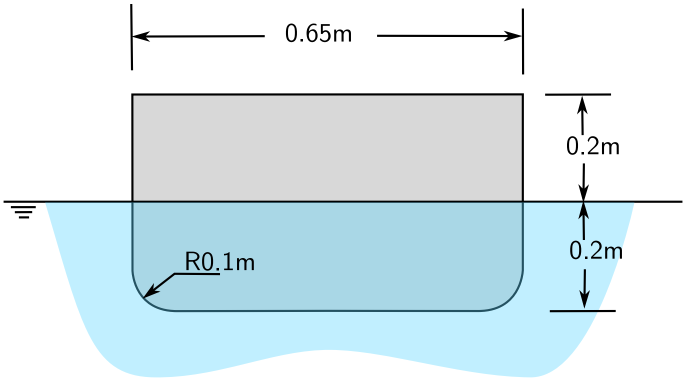

Figure 2: The model geometry and important parameters.

Table 1: Model parameters.

Cylinder radius (m)	0.325
Fillet radius (m)	0.100
Draught (m)	0.200
Volume (m^3)	0.062

The system was modelled in WAMIT 7.1, which produced a database of hydrodynamic coefficients that include first order wave forces and the main diagonal of the unidirectional QTF coefficients. This database was provided as input into ProteusDS using the RadDiffHydrodynamic feature, which made use of Newman's approximation in establishing the remainder of the QTF database. ProteusDS simulated the wave field in the time domain and forces acting on the rigidbody were computed.

A total of six simulations were conducted, replicating three monochromatic wave tests from [2] and three bichromatic waves from [3]. All conditions are deep water waves and the conditions are summarised in Tables 2 and 3, respectively. The results of the three monochromatic wave cases are provided in Section 4.3 and the results from the three bichromatic wave cases are provided in Section 4.4.

Table 2: The wave period and amplitude for the three monochromatic wave cases.

Case id	τ (s)	A (m)
1	1.10	2.2
2	1.35	2.9
3	2.00	4.7

Table 3: The wave periods and amplitudes for the three bichromatic wave cases.

Case id	τ_1(s)	τ_2(s)	A_1 (m)	A_2 (m)
4	1.55	1.38	3.4	3.0
5	0.72	0.68	1.1	1.0
6	1.55	1.13	1.13	3.4

4.3 Monochromatic waves

Table 4 presents a comparison of the expected and simulated non-dimensionalised first order wave loads. The expected loads were taken from plotted data in [2]. The first order wave forces F output by ProteusDS were non-dimensionalised as:

$$F_{nd} = \frac{F}{\rho g A L^2} \tag{8}$$

where F_{nd} is the non-dimensionalised force, ρ is the fluid density, g is the gravitational acceleration, A is the wave wave amplitude and L is the characteristic length, set to 1m.

Table 4: A comparison of the expected and simulated first order wave forces.

	Surge first order force			Heave first order force		
Case ID	expected	simulated	% diff	expected	simulated	% diff
1	0.17	0.17	0%	0.10	0.09	11%
2	0.14	0.15	7%	0.14	0.14	0%
3	0.08	0.08	0%	0.22	021	5%

Table 5 presents a comparison of the expected and simulated non-dimensionalised steady wave drift loads. The expected loads were taken from plotted data in [2]. The dimensional wave drift forces output by ProteusDS were non-dimensionalised as:

$$F_{nd} = \frac{F}{\rho g A^2 L} \tag{9}$$

Table 5: A comparison of the expected and simulated mean wave drift forces.

	Surge steady wave drift force			Heave steady wave drift force		
Case ID	expected	simulated	% diff	expected	simulated	% diff
1	0.17	0.16	-6%	0.17	0.16	-6%
2	0.10	0.10	0%	0.19	0.19	0%
3	0.02	0.02	0%	0.15	0.14	-7%

The simulations results matched expected values closely. The size of the percent differences between expected and simulated comes from limited precision in data provided and round-off error because of the small magnitudes.

4.4 Bichromatic waves

Table 6 presents a comparison of the expected and simulated non-dimensionalised slowly varying wave drift loads. The expected loads were taken from plotted data in [3]. The dimensional wave drift forces output by ProteusDS were non-dimensionalised as:

$$F_{nd} = \frac{F}{\rho g A^2 L} \tag{10}$$

where characteristic length L, was set to the cylinder diameter.

Table 6: A comparison of the expected and simulated slowly varying wave forces.

	Surge slowly varying wave force			Heave slowly varying wave force		
Case ID	expected	simulated	% diff	expected	simulated	% diff
4	0.22	0.22	0%	0.56	0.57	2%
5	0.56	0.58	4%	0.15	0.16	7%
6	0.36	0.32	-11%	0.60	0.54	10%

The simulations results matched expected values closely. The size of the percent differences between expected and simulated comes from limited precision in data provided and round-off error because of the small magnitudes.

5 Conclusion

Second order wave loading, while small compared to first order excitation and diffraction loads, can contribute important effects that cause resonance in some floating platform degrees of freedom.

The ProteusDS WAMIT database converter was extended to export wave drift QTF coefficients, and the RadDiffHydrodynamicModel feature in ProteusDS was extended to accept QTF coefficients to model slowly varying wave drift loads.

WAMIT was used to model and compute a database of hydrodynamic coefficients for the body used in the tank tests of [2] and [3]. Three of the monochromatic wave experiments and three of the bichromatic wave experiments were recreated in simulation using ProteusDS and the WAMIT coefficient model.

The forces predicted by ProteusDS matched the forces reported by [2] and [3] within 10%.

References

[1] O. M. Faltinsen. *Sea loads on ships and offshore structures*. Cambridge University Press, New York, NY, 1990.

[2] N. Fonseca, J. Pessoa, S. Maavrakos, and Marc Le Boulluec. Experimental and numerical investigation of the slowly varying wave exciting drift forces on a restrained body in bi-chromatic waves. *Ocean Engineering*, 38, 2011.

[3] J. Pessoa and J. Fonseca. Investigation of depth effects on the wave exciting low frequency drift forces by different approximation methods. *Applied Ocean Research*, 42, 2013.